Modeling confined jet flow

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The prediction of confined jet mixing, which occurs in many processes from jet pumps to furnaces, is studied by testing and improving turbulence models. Numerical simulations of axisymmetric parabolic jet flows, with the two-equation k-ε eddy-viscosity model and the second-moment closure in its algebraic form (ASM), are compared with measurements. This leads to the identification of defects that cause high rates of mixing, similar to those shown in earlier work with free jets. Modifications to the dissipation rate equation, proposed for the free jet, are addressed by examining the effects of anisotropy-related proposals and the sensitization to irrotational strains. The involvement of large structures in transport phenomena is also considered via bulk-convection-based models. A combination of 20 percent gradient diffusion and 80 percent bulk convection appears to mimic the transport process for turbulent energy reasonably well. The computations, made with a finite-difference/finite-volume parabolic solver with an efficient forward marching technique, show that there is still room for improvement in the modeling of jet flows, and some suggestions for further work are made.

Keywords: turbulence; modeling; jets; ejectors

Introduction

The use of confined jet devices is important in many engineering applications such as in combustors and ejectors, which are used as pumps, in boundary-layer control, in noise suppression, and in thrust augmentors in both conventional and V/STOL aircraft. Confined jet flow involves momentum transfer from a discharging jet to a secondary stream; it can be considered in terms of several basic interacting flows, in which the primary jet flow represents the dominant component.

The performance of computational fluid dynamics (CFD) turbulence models, when applied to jet flows, is usually assessed against the round free jet, since this bears some resemblance to the confined flow situation and has the advantages of possible self-preservation and a large volume of experimental data. The term *free jet* is used to describe a jet exhausting into ambient surroundings. The round free jet has a persistent difficulty that has been attributed by Launder *et al.* (1984) and others partially to the weaknesses of the dissipation rate equation—namely the so-called "round jet anomaly," which is the prediction, incorrectly, of a similar rate of speed for both round and plane jets. In practice, the round free jet spreads some 20 percent less rapidly than the predicted rate. More anomalously, as discussed by Fu (1988), for example, for the case of the swirling round free jet the simulated rate of spread is less than that of the swirl-free round jet, while the opposite is true in practice.

These anomalies appear to occur with all "standard" two-equation turbulence closures, such as the k-ε and algebraic stress model (ASM). They can also occur when using Reynolds stress models (RSM). Measures to cure turbulence model defects are abundant in the literature and stretch from the clearly unsatisfactory readjustment of basic constants, which affects the claim of universality of the model as demonstrated by Launder and Morse (1979), to more rational attempts to give a better representation of the spectral nature of turbulence by using two or more turbulence time and/or length scales, as has been described by Launder and Schiestel (1978). The difficulties of standard turbulence models when applied to axisymmetric free shear flows appear to be connected with (1) the existence of large-scale structures that are responsible for the shear-layer growth and entrainment; and (2) the dissipation rate equation, which is widely accepted to be conceptually weak. These phenomena occur in all types of turbulent shear flow, so improvement in CFD closure schemes, derived from examining round jets, may well produce models with better physical realism when used with other types of turbulent shear flow.

Generally, for the round free jet, attention has been focused on the dissipation rate equation. The performances of anisotropy-related modifications, incorporated in the equation, were examined by Huang, Launder, and Leschziner (1986) and Haroutunian, Ince, and Launder (1988). The implied assumption of a dominating small-eddy transport mechanism in the standard CFD models is also known to be physically invalid. The acceptance of this type of modeling stems mainly from the advantages that are brought to the numerical scheme when using two-equation models. Ribeiro and Whitelaw (1980) have found evidence from their investigations of swirling coaxial jets that the model of Bradshaw, Ferriss, and Atwell (1967) was more consistent with their results. This assumes the diffusive flux of turbulent energy to be proportional to energy times a local diffusive velocity characteristic of the large-scale motion. This "bulk convection velocity" V_k was assumed to scale on the root square of the shear stress. However, an empirical similarity form for the distribution of V_k must also be assumed. In practice, V_k should be related to the velocity fields in "typical" large eddies, but our conceptual and theoretical understanding of this linkage is too incomplete to formulate satisfactory deterministic closures on this basis.

Biringen (1978) and Biringen and Abdol-Hamid (1985) used the approach of Bradshaw *et al.* (1967) to model jet flows, and

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introduced a modified empirical function for the bulk convection velocity with negative values near the centerline and high positive values at the jet edge. This was said to be compatible with the idea that energy should be transported away from regions of maximum energy. Biringen (1978) also introduced the idea of bulk-convective transport of ε, with a bulk-convection velocity V_ε. However, his empirical forms for V_k and V_ε had no supporting experimental data for jet flows. The measurements of jet flows in a constant-area duct by Yule and Damou (1991) included data for V_k, and these are used in the present investigation.

The absence of good experimental data for the initial conditions is often regarded as an important factor causing variations in the performance of models. The present computations also benefit from the availability of the database of Yule and Damou (1991) and Damou (1988), which includes accurate descriptions of the initial conditions and early stages of flow development. The nozzle diameter was $D = 8.7$ mm and the duct radius was $r_s = 97$ mm in their experiments. Measurements were made using different values of the Craya–Curtet dimensionless parameter Ct, which is based upon integrals of mass and momentum flux across the inlet plane of the duct (which is also the outlet plane of the jet nozzle):

$$\mathrm{Ct} = \left[\frac{\dot{m}_S^2/\rho A_S}{\dot{M}_J - \dot{m}_J\dot{m}_S/\rho A_S}\right]^{1/2} \tag{1}$$

The principal flows investigated by Yule and Damou (1991) were for cases with Ct = 1.7 and Ct = 4.0. A free jet (in ambient surroundings) has Ct = 0. Yule and Damou (1991) showed that a bulk convection assumption agreed with their data, and the present work thus devotes some effort to considering a large-scale transport-related bulk convection, where the bulk-convection velocity is assumed to scale on the square root of the maximum turbulent kinetic energy at any cross section of the jet. The role of the large structures in the transport phenomena is considered by evaluating the accuracy of both a solely bulk-convection-based model and also a model combining partial contributions from both gradient diffusion and bulk convection: a combined bulk-convection/gradient-diffusion model (BC/GD). A comparative study of the most important dissipation source-related modifications is also made. This paper thus has the objectives of testing previous model modifications, made on the basis of round free jet comparisons, in the confined jet situation and also of testing the bulk-convective and BC/GD-closure assumptions by making use of recent extensive turbulence data for this type of flow.

The standard models

The two-equation k-ε model, which specifies the turbulent viscosity, and the algebraic stress model (ASM), which takes into account the stress transport, are used with boundary-layer approximations of the equations, which are legitimate for these jet flows that lack recirculation zones and are thus parabolic in nature.

The two-equation k-ε model

As described by Jones and Launder (1972), for example, the eddy-viscosity concept of Prandtl–Kolmogorov relates turbulent stresses to mean strains in a Boussinesq "gradient diffusion" form using an eddy viscosity ν_t that scales on velocity and length scales $k^{1/2}$ and l_S so that $\nu_t = C_\mu k^{1/2} l_S$ and $\varepsilon = k^{3/2}/l_S$ so that $\nu_t = C_\mu k^2/\varepsilon$. The closure is achieved by solving transport equations for k and ε (together with the Reynolds momentum equation), where

$$\frac{Dk}{Dt} = \frac{\partial}{\partial x_k}\left[\left(\frac{\nu_l}{\sigma_{k,l}} + \frac{\nu_t}{\sigma_{k,t}}\right)\frac{\partial k}{\partial x_k}\right] + \nu_t\left(\frac{\partial U_i}{\partial x_k} + \frac{\partial U_k}{\partial x_i}\right)\frac{\partial U_i}{\partial x_k} - \varepsilon \tag{2}$$

$$\frac{D\varepsilon}{Dt} = \frac{\partial}{\partial x_k}\left[\left(\frac{\nu_l}{\sigma_{\varepsilon,l}} + \frac{\nu_t}{\sigma_{\varepsilon,t}}\right)\frac{\partial \varepsilon}{\partial x_k}\right] + C_{\varepsilon 1}\frac{\varepsilon}{k}\nu_t\left(\frac{\partial U_i}{\partial x_k} + \frac{\partial U_k}{\partial x_i}\right)\frac{\partial U_i}{\partial x_k} - C_{\varepsilon 2}\frac{\varepsilon^2}{k} \tag{3}$$

Notation

A_J	Cross-sectional area of nozzle
A_S	Cross-sectional area of confining duct
b	Jet half-width; radial distance to position where $U = U_2 + (U_{CL} - U_2)/2$
C_μ	Basic coefficient in gradient diffusion assumption
Ct	Craya–Curtet parameter
D	Nozzle diameter
k	Turbulent energy; $k = \frac{1}{2}(\overline{u^2} + \overline{v^2} + \overline{w^2})$
k_m	Maximum turbulent energy
l_S	Turbulence length scale
$\dot{m}_J$	Mass flow rate through nozzle
$\dot{m}_S$	Mass flow rate through inlet of confining duct
$\dot{M}_J$	Momentum flow rate through nozzle
$\dot{M}_S$	Momentum flow rate through inlet of confining duct
P	Mean static pressure
P_k	Rate of production of k
P_{ij}	Production tensor of $\overline{u_i u_j}$
p	Fluctuating component of static pressure
r	Radial distance from centerline of ducted jet
r_S	Radius of duct
S_Φ	Source term of general variable Φ
U	Mean velocity component in x-direction
U_i	"ith" component of mean velocity
U_{CL}	Centerline mean velocity
U_J	Jet exit (centerline) velocity
U_s	Secondary stream velocity at duct inlet
U_2	Secondary stream velocity, distance x downstream
u, v, w	Longitudinal, radial, and azimuthal components of fluctuating velocity
u_i	"ith" component of fluctuating velocity
$\overline{uv}_m$	Maximum shear stress at a jet cross section
V	Mean velocity component in the r direction
V_k	Bulk convection velocity for k
V_ε	Bulk convection velocity for ε
x	Longitudinal distance from nozzle exit plane
x_i	"ith" component of position vector

Greek symbols

α	Proportion of turbulent diffusion due to bulk convection
ε	Rate of dissipation of turbulent energy, per unit volume of fluid
δ_{ij}	Kronecker delta
ν_t and ν_l	Kinematic turbulent (or eddy) and laminar viscosity
ρ	Fluid density
Φ	General scalar variable

Table 1 The constants of the two-equation k-ε model

C_μ	$C_{\varepsilon 1}$	$C_{\varepsilon 2}$	$\sigma_{k,l}$	$\sigma_{k,t}$	$\sigma_{\varepsilon,l}$	$\sigma_{\varepsilon,t}$
0.09	1.44	1.92	1.0	1.0	1.0	1.25

Table 1 gives commonly accepted values for the coefficients in these equations. Turbulent diffusion of k, the first term on the right-hand side of Equation 2, is modeled by a gradient diffusion, $\overline{pv}$, which occurs in the full equation, is not explicitly modeled. Equation 3 also models the diffusion of ε by using a gradient diffusion assumption.

The algebraic stress model (ASM)

The ASM is a procedure for solving transport equations for all individual Reynolds stresses. The variant of Rodi (1972) assumes proportionality between the net transport of each stress and the corresponding transport of k, which yields

$$\frac{\overline{u_i u_j}}{k}(P_k - \varepsilon) = (1 - C_2)P_{ij} + \tfrac{2}{3}C_2\,\delta_{ij}P_k - C_1\frac{\varepsilon}{k}\overline{u_i u_j} + \tfrac{2}{3}(C_1 - 1)\delta_{ij}\varepsilon \tag{4}$$

where $C_1 = 1.8$ and $C_2 = 0.6$. Substituting expressions for P_{ij} into Equation 4 yields a set of algebraic equations that relates the Reynolds stresses, $\overline{u^2}$, $\overline{v^2}$, $\overline{w^2}$, and $\overline{uv}$.

The wall treatment

The economical "wall functions" method outlined by Launder and Spalding (1974) is used here. This approach is adequate for the ducted jet flows of Yule and Damou (1991), for which the initial boundary-layer thickness was less than 2 percent of the duct diameter.

The numerical procedure

The general equations for steady axisymmetric flow of the boundary-layer type reduce to the following forms:

Continuity:

$$\frac{\partial \rho U}{\partial x} + \frac{1}{r}\frac{\partial \rho r V}{\partial r} = 0 \tag{5}$$

Momentum:

$$\frac{\partial \rho U^2}{\partial x} + \frac{1}{r}\frac{\partial r\rho U V}{\partial r} = -\frac{dP}{dx} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial U}{\partial r} - r\rho\overline{uv}\right) \tag{6}$$

$$\frac{\partial \rho U\Phi}{\partial x} + \frac{1}{r}\frac{\partial \rho r V\Phi}{\partial r} = -\frac{1}{r}\frac{\partial}{\partial r}\left(-r\Gamma_\Phi\frac{\partial \Phi}{\partial r} + r\rho\overline{\Phi v} + S_\Phi\right) \tag{7}$$

Equation 7 can represent Equations 2 or 3 by choosing $\Phi = k$ and $\Phi = \varepsilon$, respectively, and inserting gradient diffusion models for $\overline{kv}$ and $\overline{v\varepsilon}$.

In the PASSABLE code of Leschziner (1981) that has been employed here, the general equations are discretized using the finite-control volume approach. A nonuniform orthogonal grid is used, with more nodes near the wall and in the central jet zone. The axial velocity component U and scalars P, k, and ε have a common control volume. The radial mean-velocity component V is staggered midway between nodal locations, which provides advantages for the formulation of the continuity equation from which V is obtained. Approximations are made to yield a set of algebraic equations that can be illustrated for Ψ_j, which represents a vector whose components consist of the nodal values at the downstream location:

$$A_j^d\Psi_j^d = A_j^u\Psi_j^u + A_{j+1}^d\Psi_{j+1}^d + A_{j-1}^d\Psi_{j-1}^d + B_j \tag{8}$$

where A_j are flux coefficients that consist of a combination of convective and diffusive contributions, and B_j represents the source term, which is made to adopt a linear form to promote the stability of the solution algorithm. The hybrid difference scheme (HDS) was used as described by Leschziner (1981).

The solution of the momentum equation requires knowledge of the pressure gradients. The treatment in the code uses, initially, the pressure gradient of the upstream station. This results in a velocity distribution that will not necessarily satisfy overall mass conservation. The aim is therefore to iterate and reduce to zero the amount of mass flux imbalance. The finite-difference equations for the variables U, k, and ε are solved by the tridiagonal matrix algorithm (TDMA) in the form of a transformation of the set of equations into a general recurrence relation for Ψ_j. Approximately 60 nodes across the flow, from the centerline, were found to give grid-independent solutions. All computations reported here were carried out using 80 nodes, also, approximately 1,000 steps downstream were taken to cover the flow regime.

Modifications of the standard models

Dissipation source related modifications

The dissipation source term is the second term on the right-hand side of Equation 3. Several modifications have been used and applied to the round free jet:

(1) The correction of Pope (1978) assumes an extra source for ε (which is zero for plane flows):

$$\frac{C_\varepsilon^P}{4}\left(\frac{k}{\varepsilon}\right)^3\left(\frac{\partial U}{\partial r} - \frac{\partial V}{\partial x}\right)^2\frac{V}{r} \tag{9}$$

where

$C_\varepsilon^P = 0.79$

(2) Hanjalic and Launder (1980) added the term

$$\varepsilon_{ijk}\varepsilon_{lmk}k\frac{\partial U_i}{\partial x_j}\frac{\partial U_l}{\partial x_m} \tag{10}$$

(3) Khajeh-Nouri and Lumley (1973) replaced the generation of k, which occurs in the ε source term, with a term containing the second invariant of the anisotropy, so that the generation term becomes

$$C_\varepsilon^{lk}A_2\frac{\varepsilon^2}{k} \tag{11}$$

where $C_\varepsilon^{lk} = 4.27$ and A_2 represents the second invariant of the isotropy tensor defined as $A_2 = (\overline{u_i u_j} - \tfrac{2}{3}\delta_{ij}k)^2/k^2$. This modification is applied to the ASM model.

(4) Lumley and Zeman (1979) allowed a share in the determination of the generation of ε, between the generation of k and the anisotropy-dependent contribution. The modified term was

$$C_{\varepsilon 1}^{Z1}P_k^\varepsilon k + C_{\varepsilon 2}^{Z1}\frac{A_2}{1 + 1.5A_2}\frac{\varepsilon^2}{k} \tag{12}$$

where

$C_{\varepsilon 1}^{Z1} = 0.475$ and $C_{\varepsilon 2}^{Z1} = 5.46$

Accounting for the large eddies in turbulent transport

All standard models assume, at critical points in their closures, the representation of turbulent diffusion by a gradient diffusion process. This incorrectly implies that the small-scale eddies dominate turbulent transport. Attempts to remove this physical shortcoming have been made based on the definition of a bulk-convection velocity V_k which is attributable to the "sweeping" transport by the large eddies. For the transport of k, one has

$$\overline{kv} = CV_k k \tag{13}$$

where C is usually consolidated in V_k and taken as unity. Yule and Damou (1991) proposed

$$V_k = k_m^{1/2} f(r/b) \tag{14}$$

Figure 1 shows that data for Ct = 1.7 and $f(r/b)$ (i.e., $\overline{kv}/kk_m^{1/2}$) are found to tend towards a similarity form, which is the same for the different jet flows.

Bradshaw *et al.* (1967) assumed a scaling on $(\overline{uv})_m^{1/2}$, rather than $k_m^{1/2}$, when considering bulk-convective transport in a boundary layer. Biringen (1978) followed this example when considering round jets in constant velocity streams. The distribution of V_k of Biringen was not based directly upon measurements, and as can be seen in Figure 1, it does not agree well with jet data.

Equation 13 assumes that all diffusion of k is by the bulk-convective mechanism. In reality one should envisage contributions from both bulk-convective (large-eddy-related) diffusion and gradient (small/medium-eddy-related) diffusion. The partitioning of diffusion between the two processes has been examined by using the following approximation for combined BC/GD. This hypothesis has been discussed by several authors in the past, but it has seldom been utilized:

$$\overline{kv} = \alpha CV_k k + (1-\alpha)C_\mu k^2 \varepsilon^{-1} \partial k/\partial r \tag{15}$$

Thus, for example, a constant value $\alpha = 0.8$ throughout the flow corresponds to 80 percent of diffusion by convective transport and 20 percent by gradient diffusive transport, with no changes in turbulent energy spectrum. This 20:80 percentage ratio between gradient diffusive and bulk-convective diffusion was also assumed by Biringen and Abdol-Hamid (1985); however, there is no firm justification for a fixed ratio for all jets nor, indeed, for all positions in one jet. The 20:80 percentage ratio does appear to have a degree of realism, since the spectra of Damou (1988) show a similar ratio of the energy content of turbulence length scales equal to 1/20th of the jet half-width. Ideally a two-length-scale/two-energy-scale approach would seem to be called for, in which the closure hypotheses for the two mechanisms have their own scaling parameters. This would be a development of the two-scale closure described by Launder and Schiestel (1978) and Schiestel (1983), and the effect would be an approximate modeling of changes in the shape of the wave number spectrum of the turbulent energy. The division between gradient and bulk-convective transport would thus be provided by solution, rather than by assumption.

Results

The initial conditions of the ducted jets were derived from the measurements of U, $\overline{uv}$, and k of Yule and Damou (1991). In the high-velocity gradient region near the jet nozzle, where cross-wire hot-wire probes are inaccurate, it was assumed $k = (3/2)\overline{u^2}$. However, $\overline{uv}$ is known in this region, due to the jet exit flow being a fully developed turbulent pipe flow. Figure 2 gives an illustration of comparisons between predictions and data for the streamwise evolution of the centerline velocity U_{CL} for the ducted jet with Ct = 1.7, for which $U_J = 144$ m/s and $U_S = 11.4$ m/s. The results of the k-ε model in Figure 2 indicate the prediction of too much mixing with an overprediction of both the velocity decay and rate of spread for the unattached jet. This is similar to the round jet anomaly observed for the free round jet. Downstream of approximately $x = 5r_S$ (i.e., $x = 55D$), the jet spreading is influenced by the duct wall

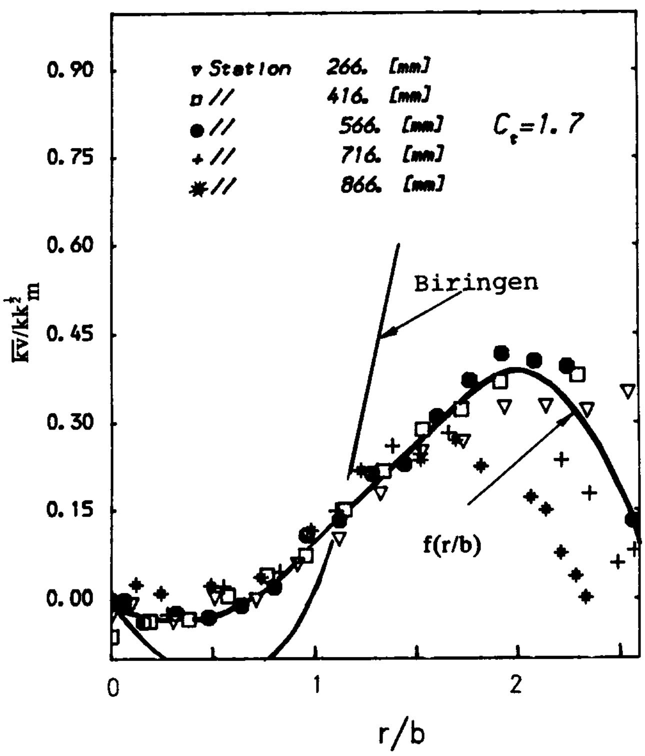

Figure 1 Normalized bulk convection velocity for Ct = 1.7, the assumed distribution for $f(r/b)$, and the distribution based upon Biringen's (1978) assumption for V_k. From Yule and Damou (1991)

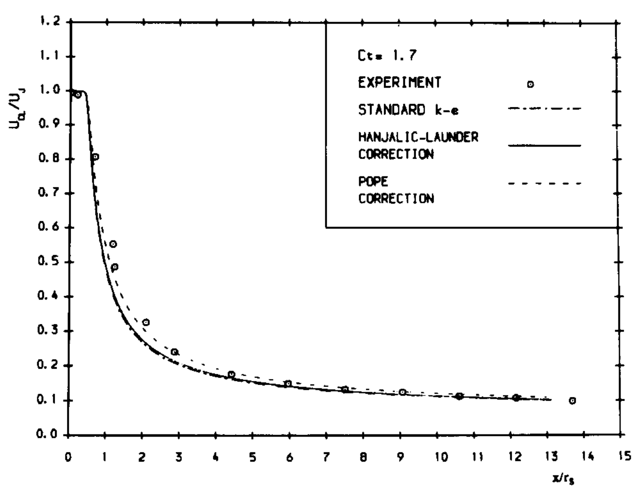

Figure 2 Measured and predicted centerline velocity decay for ducted jet with Ct = 1.7, including standard k-ε model, Pope's (1978) modification, and Hanjalic and Launder's (1980) modification

boundary layer and the k-ε prediction becomes excellent as the flow tends towards a pipe flow. The inclusion of the correction of Hanjalic and Launder (1980), as seen in Figure 2, appears to offer a marginal improvement of the prediction of the unattached jet when compared with the velocity measurements. This is in accordance with the relative absence of strong normal strain rates for this ducted jet compared with the round free jet. One may conclude that the "irrotational deformation" effect is not a major source of the round jet anomaly.

As can be seen in Figure 2, the modification of Pope (1978) yields a slight overcorrection that implies less mixing than the experimental observation. This also agrees with the results of Huang (1986), who applied Pope's modification to the free jet. There is also some slight underestimate of the velocity decay as the jet-wall boundary-layer interaction commences, around $x \simeq 55D$ ($x \simeq 5r_S$). It is clear, however, that Pope's modification gives a very reasonable prediction of the centerline velocity decay of the ducted jet for Ct = 1.7, and a similarly good agreement was found with the other main jet flow of Yule and Damou (1991) with Ct = 4.0.

Figure 3 makes a further comparison of the data for Ct = 1.7, with the ASM model and also the modified ASM models of Lumley and Khajeh Nouri (1973) and Zeman and Lumley (1979). The unacceptable performance of the ASM model in this situation is highlighted in Figure 3, where a greater overprediction of jet decay than for the k-ε model can be seen. This is in accord with ASM predictions of the round free jet in still surroundings, which have also been found to be poor. The two Lumley modifications both overpredict the unattached jet decay initially; however, they then quite rapidly adjust to reasonable predictions beyond $x \simeq 6r_S$, i.e., $x \simeq 65D$.

In these ASM modifications the inclusion of anisotropy in the dissipation rate equation produces numerically related difficulties. The decoupling of the turbulence energy and its rate of dissipation produced a numerical weakness that materialized as solution convergence instabilities. Consequently, some measures were taken to circumvent these difficulties: (1) limiting the level of anisotropy permitted in the early stages of the flow development; and (2) increasing the degree of implicitness in the solution by using strong underrelaxation factors and augmenting the number of in-step iterations. Both anisotropy corrections yield an initial, even faster decay of the centerline velocity than both the standard ASM and the standard k-ε models. However, the modified models predict less

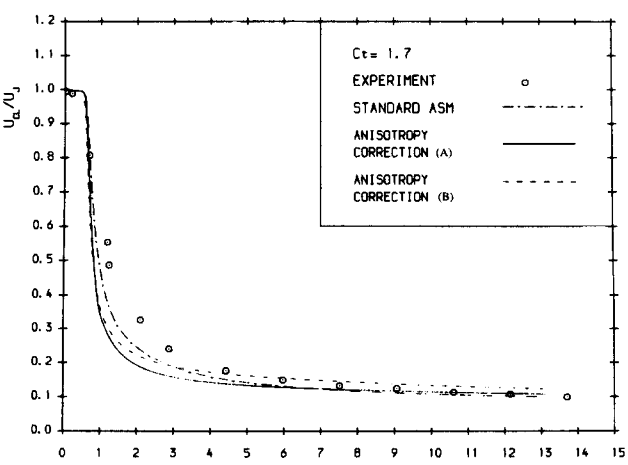

Figure 3 Measured and predicted centerline velocity decay for ducted jet with Ct = 1.7, including standard ASM, Lumley and Khajeh Nouri's (1974) modification (A), and Zeman and Lumley's (1979) modification (B)

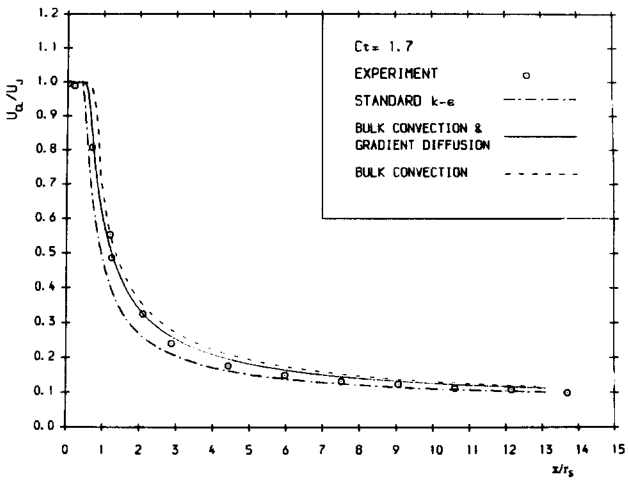

Figure 4 Measured and predicted centerline velocity decay for ducted jet with Ct = 1.7, including standard k-ε model, "pure" bulk-convection model (Equation 13), and BC/GD model (Equation 15)

rapid mixing than the experiment in the downstream region of the flow. Comparatively, the correction of Zeman and Lumley (1979) is slightly closer to the experimental observations. However, one may conclude from the failure of these models to predict the ducted jet flow, as well as the free jet flow, that anisotropy effects are not the principal reasons for differences between measured and computed jet flows, at least when such effects are implemented in an ASM model.

As shown in Figure 4, the large-eddy-related model that uses the "pure" bulk-convection hypothesis for the diffusive transport of k, i.e., Equations 13 and 15, predicts slightly smaller rates of mixing than the experimental values for $x > 1.5r_S$, i.e., $x > 17D$. As can be seen in Figure 4, although the bulk-convection model is very much more accurate than the standard k-ε model near the nozzle, it produces increasingly less accurate predictions far downstream as the flow tends towards a pipe flow. Figure 4 also includes the predictions of the combined BC/GD assumption for $\overline{kv}$, as given by Equation 15 with the value $\alpha = 0.8$; i.e., 80 percent of diffusive transport of k is assumed to be due to bulk convection. It can be seen that the BC/GD prediction has an excellent degree of accuracy throughout the jet length and that it is, overall, of similar accuracy to the best of the dissipation equation modifications, i.e., Pope's (1978) modification. Similar results are obtained when the predictions of the various modified models are compared with the other principal ducted jet flow of Yule and Damou (1991) with Ct = 4.0, for which $U_J = 115$ m/s and $U_S = 19.0$ m/s. Figure 5 shows comparisons between predictions and measurements for this flow and, as for the Pope modification, the only discrepancy is a very slight underestimate of the rate of decay of the jet in the far downstream pipe flow zone. The jet with Ct = 4.0 is a "weak" jet flow, with relatively low excess velocity in the duct. It thus should have some similarity with an axisymmetric wake. Both the Pope and BC/GD models thus appear to perform reasonably across the full spectrum of strong and weak confined jets.

Only the centerline velocity decay has been shown here, and this could in some cases be misleading: for example, if a model does not correctly predict the shape of the velocity profile. This is the case for the ASM model, for which the predicted jet velocity profile is excessively flat in the central part of the jet. Figure 6 shows the mean velocity profiles at $x = 116$ mm and $x = 266$ mm measured by Yule and Damou (1991) and the

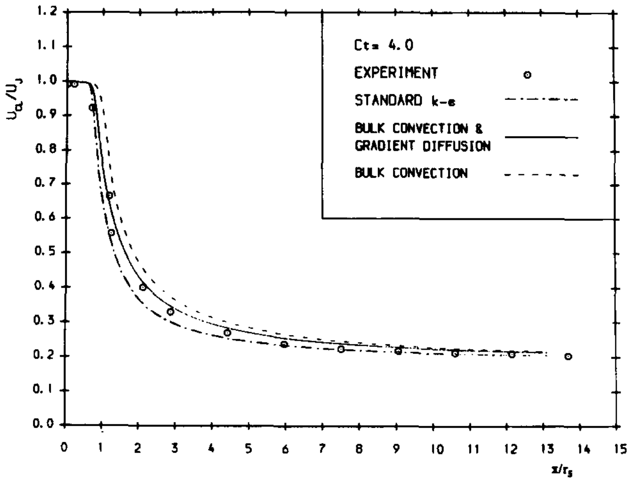

Figure 5 Measured and predicted centerline velocity decay for ducted jet with Ct = 4.0, including standard k-ε model, "pure" bulk-convection model, and BC/GD model

predictions of the standard k-ε model and the BC/GD k-ε model. The latter model is seen to provide a very good prediction of U for the complete width of the jet. The standard k-ε model provides a good prediction of the shape of the velocity profile across the jet, but not the rate of spread. It is also seen that the k-ε model gives an adequate prediction of the wall boundary layer for the flow. No attempt has been made to apply the "corrected" models outside the confines of the jet flow so that the "standard" models are applied in the secondary flow and wall boundary layer. The empirical form for V_k has been derived from data for unattached jets. Thus a problem arises when jet-duct boundary-layer interaction occurs downstream in the duct. In the absence, as yet, of a "universal" approach to bulk-convective modeling, it was necessary to "switch" from the bulk-convective scheme to the standard model where direct interaction occurred.

The jet "half-width" b is an important parameter for characterizing the jet. Figure 7 shows the measurements of b, for the two ducted jet flows, compared with the predictions of the standard k-ε model and the BC/GD and Pope modified models. It is seen that the Pope modification is in excellent agreement with all the data. The BC/GD modification provides somewhat less accurate predictions of b, but these are shown to be very much better than the standard k-ε results.

Discussion

It is clear that the ducted jet flows offer similar problems for standard CFD codes as those presented by the round free jet. It is also clear that of the various modifications that have been considered (most of which are optimized to provide agreement with the round free jet), the mean vortex-stretching modification of Pope (1978), Equation 9, and the combined BC/GD approach, Equation 15, provide the best agreement between prediction and experiment for the ducted jets. It is not the case, of course, that only one of these approaches should be correct. Indeed, both approaches attempt to include, within the equations, submodels of phenomena that are known to exist; they should thus produce increased physical realism, which is an essential objective in CFD model development. One may thus propose that some combination of both modifications should be included in the equations, although the experimental database is insufficient to propose such a combination with any degree of confidence.

The Pope modification is quite straightforward to implement; however, the user must ensure that the computer code gives zero value to the term in plane flows. The BC/GD modification must, if its underlying hypothesis is correct, be applicable to all types of turbulent shear flow, and it is here

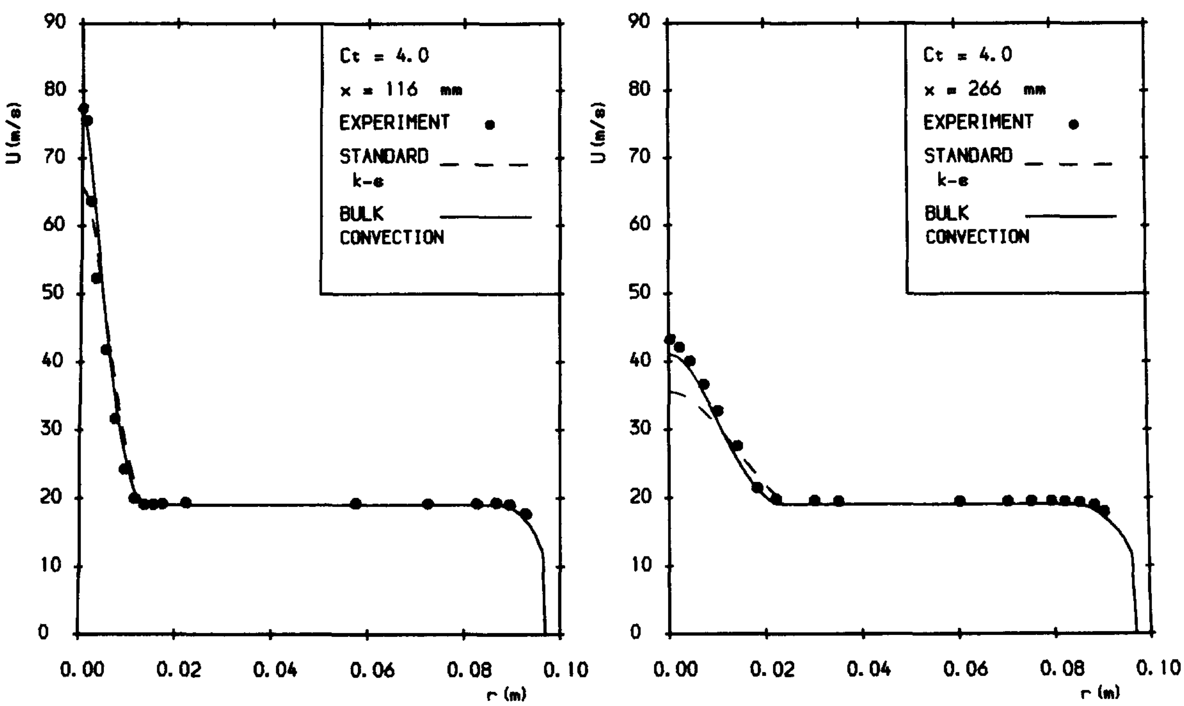

Figure 6 Comparison between predicted and measured mean velocity profiles for ducted jet with Ct = 1.7

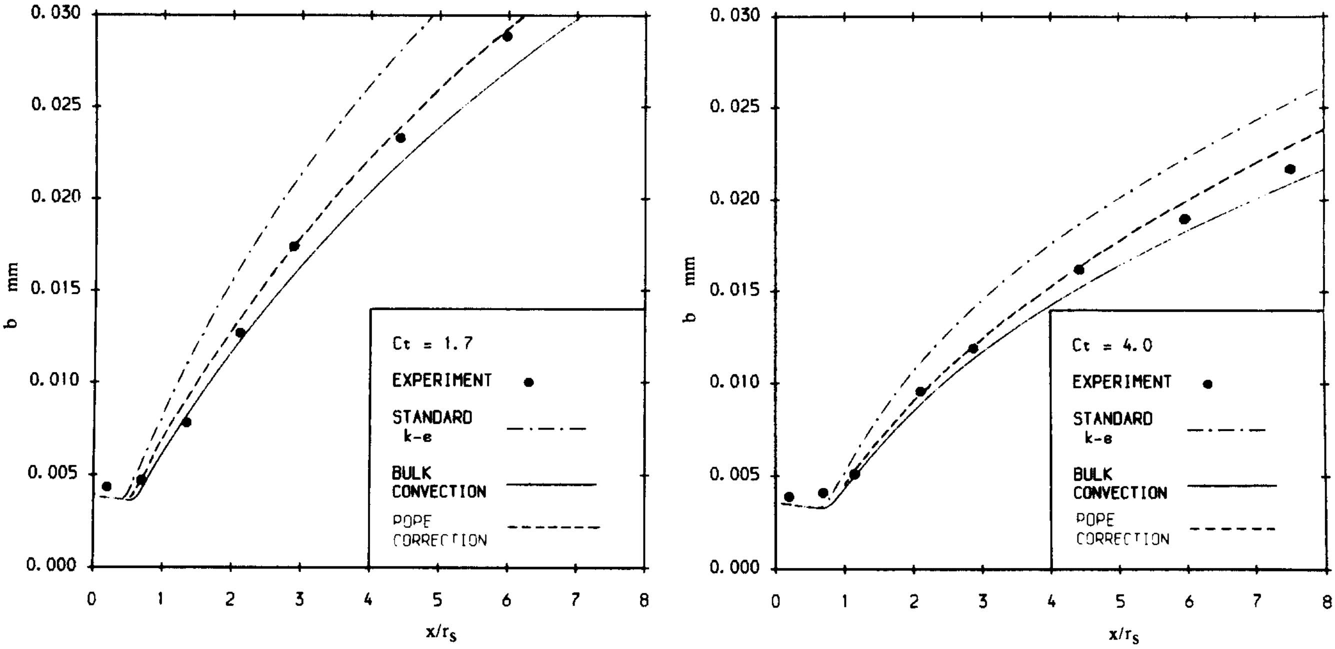

Figure 7 Measurements and predictions of the half-widths of the ducted jets

that very much more conceptual and data analysis work is required. The combined BC/GD hypothesis was found to have further advantages over the closure using "pure" bulk convection due to better numerical stability and the need for fewer iterations.

The form of V_k, represented by the function $f(\eta)$, has been deduced from measurements of $\overline{kv}$ and k for the ducted jets. It should be noted that, when implemented in the computations, the function was written for convenience in the form $f(r/l_S)$ rather than $f(r/b)$: in practice, l_S ($= \varepsilon^{-1}k^{3/2}$) has insignificant radial variation across the jet. Two further important tests were made of the bulk-convective assumption. In the first, the BC/GD-modified k-ε model was used to predict the downstream zone of a self-preserving round free jet, using the distribution of V_k found for the ducted jets. A spreading rate of $db/dx = 0.085$ was found, which compares well with the value $db/dx = 0.086$ measured by Wygnanski and Fiedler (1969). This is a significant improvement over the value $db/dx = 0.119$ predicted by the standard k-ε model. The second test involved a comparison with the case of a plane jet. The same distribution of V_k was used to predict this case, and a value $db/dx = 0.109$ was found that compares well with the value $db/dx = 0.110$ given by Rodi (1972). It is noted that there are insufficient published data for the plane jet to verify that $f(r/b)$ is similar for the round and plane jet cases.

These results show the promising nature of the ideas behind bulk-convective closures. However, problems remain, one being the empirical nature of the shape of the bulk-convection velocity distribution. A satisfactory closure should relate this distribution in a deterministic and physically meaningful way to parameters of the local flow. It is possible that the effects of intermittency and, perhaps, the encroachment of large eddies, with opposite sign, from one side of the jet to the other must be considered in such a formulation. A further difficulty is brought about by the need to formulate bulk-convective closures for other terms in the equations, including $\overline{uv}$ and the equivalent term for the ε equation. Biringen (1978) utilized an empirical bulk-convective closure for the ε equation, but once again the shape of the bulk-convection velocity profile requires justification. There is no intrinsic reason why the bulk-convection velocities for momentum, turbulent energy, and dissipation rate should be equal, and the linkage between bulk convective assumptions for the three dependent variables should be some average velocity pattern for the larger eddies, which are principally responsible for "diffusive" transport. This interesting area is in need of considerable experimental and modeling investigation before a satisfactory bulk-convective closure is achieved.

Conclusions

The standard k-ε and ASM models exhibit errors in the prediction of ducted jets that are similar to those found in earlier studies of round free jets. The various published methods of correcting the round free jet/plane jet anomaly have been studied for the case of ducted jets. The vortex-stretching dissipation source correction of Pope appears to be the most successful of the different methods, and it is recommended for general use with axisymmetric jet flows, with and without confinement. A combined bulk-convection gradient-diffusion closure of the k-ε model, based upon measurements by the authors, is also found to give satisfactory predictions of the ducted jets while giving good predictions of the round free jet and also the plane free jet.

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